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LETTER TO THE EDITOR

Existence of the energy perturbation expansion in the inverse of the quark confinement coupling constant

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Abstract. The existence or otherwise of the expansion of the energy eigenvalue in the inverse of the confinement coupling constant (α) for large values of α is discussed for a class of quark confinement potentials.

The study of infinitely rising potentials within the context of a Schrödinger equation as a means of the confinement mechanism of the quarks has become a subject of considerable interest. Quigg and Rosner (1978) and Caprasse (1979) discussed in this connection various scaling laws for the class of potentials of the form

$$V(r) = ar^{\epsilon}, \tag{1}$$

$$V(r) = b \ln r/r_0. \tag{2}$$

Martin (1977), Grosse (1977) and Grosse and Martin (1978) derived general theorems regarding the level spacings for a large class of such potentials. Their results have been quite useful in interpreting qualitatively the newly discovered particle spectra of the massive quarkonium systems.

In an earlier paper (Datta and Mukherjee 1980) we have studied analytically the eigenvalue problem for the class of confinement potentials

$$V(r) = =\zeta/r + \lambda r + \beta r^2 \tag{3}$$

by the continued fraction technique. The convergence and analyticity of the Green function for the problem have been studied in detail in terms of the Coulomb coupling constant (ζ). The analyticity of the energy eigenvalue $E(\zeta)$ in ζ near $\zeta = 0$ and the existence or otherwise of the energy perturbation series have also been discussed. Since the Green function does not have a suitable representation in terms of the major confinement coupling constant (β) we were unable to study the above problem in the complex β-plane. The purpose of this Letter is to obtain an interesting result in terms of the confinement coupling constant from our earlier results, making use of a scaling argument due to Symanzik (Simon 1970). The result, however, holds for a more restricted class of potentials (see equation (4)) than is given by (3).

We consider the potential (3) with a convenient choice of the confinement parameter,

$$V(r) = -\zeta/r + \alpha^2 r + \alpha^4 r^2, \tag{4}$$

and derive the Symanzik scaling law.

Theorem. Let $\alpha > 0$, $\lambda > 0$ and ζ real. Then the energy eigenvalue $E(\zeta, \alpha)$ satisfies the relation

$$E(\zeta, \alpha) = \lambda^2 E(\zeta\lambda^{-1}, \alpha\lambda^{-1}). \tag{5}$$

In particular

$$E(\zeta, \alpha) = \alpha^2 E(\zeta\alpha^{-1}, 1). \tag{6}$$

To prove the theorem, we start with the radial Schrödinger equation ($2\mu = \hbar = 1$)

$$-\frac{d^2}{dr^2}R + \left(-\frac{\zeta}{r} + \alpha^3 r + \alpha^4 r^2 + \frac{l(l+1)}{r^2}\right)R = E(\zeta, \alpha)R \tag{7}$$

and obtain, under a scaling transformation $r \to r/\lambda$, $p \to \lambda p$, $\bar{R}(r) - R(r/\lambda)$, the equation

$$-\frac{d^2}{dr^2}\bar{R} + \left(-\frac{\zeta\lambda^{-1}}{r} + \alpha^3\lambda^{-3}r + \alpha^4\lambda^{-4}r^2 + \frac{l(l+1)}{r^2}\right)\bar{R} = \lambda^{-2}E(\zeta, \alpha)\bar{R}. \tag{8}$$

Since $\bar{R}$ is an eigenfunction of the operator on the LHS of equation (8), we obtain the desired equation (5).

We remark that equation (5) remains valid also for complex values of ζ, α and λ for which the function $E(\zeta, \alpha)$ can be continued analytically. Since the analyticity of $E(\zeta, 1)$ at $\zeta = 0$ has already by proved in our earlier paper, the equation (6) shows the following.

Corollary. $E(\zeta, \alpha)$ for any fixed real ζ has a convergent expansion in the inverse of the confinement coupling constant (α) for large values of α.

We see that equation (5) holds also for the potential

$$V(r) = -\zeta/r + \alpha^4 r^2.$$

But the corresponding confinement coupling constant expansion will diverge since the analyticity of $E(\zeta, 1)$ in this case can be obtained only in a cut neighbourhood of $\zeta = 0$.

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